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Principal Component and Factor Analysis of Macroeconomic Indicators

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Abstract:Although macroeconomic variables have long been studied, many economists and researchers have often neglected the preliminary study of the variables regarding their similarities and differences. This paper adopted the principal component and factor analysis to assess nine macroeconomic variables by finding out the level of redundancy among them from the correlation matrix and grouping indicators with higher similarities into the same factors. Scree plots of principal components suggest grouping indicators into three factors. We then estimated the unrotated and rotated factor loadings withboth PCAand MLE estimation methods. Results from the PCA were gratifying since we obtained extreme loadings after the Varimax rotation. Out of the nine macroeconomic indicators, sixindicator were loaded on factor one, two were loaded on factor two and one was loaded on factor three.

Keywords: Macroeconomic, Principal Component, Factor Analysis, Correlation Matrix, Scree Plot, Maximum Likelihood, Loadings, Rotation, Varimax

Abbreviations: PCA – Principal Component Analysis
MLE – Maximum Likelihood Estimation

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I. INTRODUCTION

In recent times, several statistical and econometric tools are employed in the analysis of multivariate data. There is an expansive literature on applied multivariate statistical methods of data analysis and reduction namely, principal component analysis (PCA), factor analysis (FA), grouping, classification, clustering and so on. Fairly, it is often difficult to identify a complete number of random variables or features explaining a particular existing phenomenon at a given period of time. For this, the researcher includes all plausible random variables at his disposal to explain the phenomenon as best as possible where different random variables have different impacts on the phenomenon under study. It becomes computationally burdensome and time consuming analyzing multivariate data with a myriad ofrandom variables. So we adopt at least onemultivariate statistical method of data reduction to shrink the data (reduce the number of random variables) without losing a substantial amount of information captured by the original data. This ideally reduces the tasks involved in going through computational trauma.

With this effect, the paper resorted to the factor and principal component methods of data reduction which classify random variables into factors representing the entire data without a significant loss of information. Factor analysis lessens the level of redundancy existing in the data by grouping random variables with very high similarities into the same factors with their corresponding loadings. Also, relying on the fact that no substantial amount of information is lost after reduction or factoring, the principal component and factor analysis remain the motivating tools to adopt in shrinking big data.

The paper in its singularity explored the two traditional methods in analyzing data on nine macroeconomic variables. Preceding the factor analysis was the principle component which was estimated to present a fair view of the number of factors to be considered. Three components explained slightly above 80% and had standard deviations above unity from the scree plots. This suggests that, factoring the nine macroeconomic variables into three groups could lead to roughly 20% of the variances in original data unexplained which can still result into making the right reckonings and conclusionsabout the original data. The paper employed the maximum likelihood and the principal component methods of estimating the factor loadings in the factor analysis. Results from the maximum likelihood were quite striking since it fails to provide extreme loading on Inflation given the method's popularity in estimating factor loadings. The principal component approach was then used to rightly factor Inflation among the three factors.

The rest of the paper is divided into three broad sections, section II talks about the econometrics of principal component and factor analysis, section III analyses the data and provides discussionand section IV concludes.

II. METHODOLOGY

A. The econometrics of principal component

Consider a random vector of interest $X' = (X_1, X_2, \dots \dots, X_n)$ with a covariance matrix $\sum$ and eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \dots \dots \geq \lambda_n \geq 0$.

We have the linear combinations as follows:

$$Y_1 = a_1' X = a_{11}X_1 + a_{12}X_2 + \cdots + a_{1p}X_p$$

$$Y_2 = a_2' X = a_{21}X_1 + a_{22}X_2 + \cdots + a_{2p}X_p$$

$$\vdots \qquad \vdots$$

$$Y_p = a_p' X = a_{p1}X_1 + a_{p2}X_2 + \cdots + a_{pp}X_p$$

We have $Var(Y_i) = a_i' \sum a_i$ and $Cov(Y_i, Y_k) = a_i' \sum a_k$ where $i, k = 1,2, \dots, p$

The principal components $Y_1, Y_2, \dots, Y_p$ should therefore capture much information as possible.

Let $\sum$ be the covariance matrix with the eigenvalue eigenvector pairs $(\lambda_1, e_1), (\lambda_2, e_2), \dots, (\lambda_p, e_p)$ and $\lambda_1 \geq \lambda_2 \geq \cdots \dots \dots \geq \lambda_n \geq 0$, then the i^{th} principal component is given by:

$Y_i = e_i' X = e_{i1}X_1 + e_{i2}X_2 + \cdots + e_{ip}X_p$ for $i = 1,2, \dots, p$

It is interesting to note that the variance of the i^{th} principal component is the i^{th} eigenvalue.

$Var(Y_i) = e_i' \sum e_i = \lambda_i$ and $Cov(Y_i, Y_k) = e_i' \sum e_k = 0$ for $i = 1, \dots, p$ and $i \neq k$.

Remarks: The principal components are linear combinations of the random variables. They are uncorrelated and have variances equal to the eigenvalues of $\sum$ (the covariance matrix) and their development does not require any distributional assumption about multivariate normality.

B. Proportion of variance explained

The proportion of the total variance explained by the k^{th} principal component can be written as:

$\frac{\lambda_k}{\lambda_1 + \lambda_2 + \cdots + \lambda_p}$, where λ_k is the eigenvalue of the k^{th} principal component. For instance if the first $k$principal components can explain most of the variations in the population covariance, then$k$ variables can replace the original p variables withlittle loss of information.

C. Factor analysis

The PCA and factor analysis attempt to find an approximation to the covariance matrix, where as principal component analysis is generally concerned with data reduction, factor analysis is generally concerned with whether the data are consistent with the factor model under consideration. The factor model in matrix notation can be written as:

$X - \mu = LF + \varepsilon$, where X depends linearly on unobservable common factors $F$and errorsε. The covariance matrix $\sum$ implied by the factor model can be derived as:

The assumptions of the factor model are: $E(F) = 0$ $\qquad Cov(F) = E(FF') = I$

$E(\varepsilon) = 0 \qquad Cov(\varepsilon) = E(\varepsilon\varepsilon') = \varphi \qquad Cov(\varepsilon F) = Cov(\varepsilon F') = Cov(\varepsilon' F) = 0$

$\sum = Cov(X) = E(X - \mu)(X - \mu)'$

$\sum = Cov(X) = E[(LF + \varepsilon)(LF + \varepsilon)']$

$\sum = Cov(X) = LE(FF')L' + E(\varepsilon F')L' + LE(F\varepsilon') + E(\varepsilon\varepsilon')$

$\sum = Cov(X) = LL' + \varphi$; where LL' are the factor loadings and φ is the specific variance

$Var(X_i) = l_{i1}^2 + \cdots + l_{im}^2 + \varphi_i$ and $Cov(X_i, X_k) = l_{i1}l_{k1} + \cdots + l_{im}l_{km}$

The communality is the sum of squares of the factor loadings. It is the proportion of the variance contributed by the common factors whereas the specific variance is the proportion of variance unexplained by the common factors.

$$\sigma_{ii} = l_{i1}^2 + \cdots + l_{im}^2 + \varphi_i$$

$Var(X_i) = Communality + Specific\ Variance$; $Communality(h_i^2) = l_{i1}^2 + \cdots + l_{im}^2$

D. Non-uniqueness of factor loadings

Suppose we have an orthogonal matrix T, the factor model can be written as:

$$X - \mu = LF + \varepsilon = LTT'F + \varepsilon = L^*F^* + \varepsilon$$

$E(F^*) = T'E(F) = 0$ and $Cov(F^*) = T'Cov(F)T = T'T = I$

$\sum = LL' + \varphi = LTT'L' + \varphi = (L^*)(L^*)' + \varphi$

E. Factor rotation

Often time unrotated factor loadings become a bit unspecified given the insignificant differences among factors for specific variables. This may leave us in a state in indecisiveness as to which factor to classify a particular random variable. Factor rotation is thus necessary to obtain extreme loadings that may help us in easily classifying random variables into their rightful factors. It can be achieved by the Varimax and Promax method. However, this paper only talks about the former which employs the orthogonal type of rotation of factor loadings and common factors.

$$\hat{L}^* = \hat{L}T$$

Where T is either given by:

$$\begin{cases} T = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \\ T = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \end{cases}$$

The Varimax criterion of factor rotation is given by:

$V = \frac{1}{p}\sum_{j=1}^{m}\left[\sum_{i=1}^{p}\tilde{l}_{ij}^{*\,4} - \left(\sum_{i=1}^{p}\tilde{l}_{ij}^{*\,2}\right)^2/p\right]$ where $\tilde{l}_{ij}^{*} = \hat{l}_{ij}^{*}/\dot{\hat{h}}_i$

III. DATA ANALYSIS

A. Brief description of data and estimation plan

The data used in this analysis are nine variable macroeconomic monthly data for Liberia taken from January 2006 to December 2014. The indicatorsincluded in the analysisare Inflation Rate, Government Expenditures, Real Exchange Rate, Interest Rate, Inflow, Outflow, Imports, Exports and GDP. The motivation for employing the principal component and factor analysis to these indicators is, most of these variables are often speculated to be highly related to one another and henceprecision of estimation may be questionable if we will in the near future want to determine the impact of these indicators on a particular event. The need thus to group these variables with the purported high similarities into common factors is vital.Factor analysis lessens the redundancy exhibited in the data and underscores the precision with which true parameters are estimated when factor scores are formed. We would clearly learn in the outputs from ***R statistical software package*** how we can conduct first, the principal component and then the factor analysis of these macroeconomic indicators adopting the two conventional methods of estimation (the principal component and the maximum likelihood methods of estimation)

It is worth transforming the covariance matrix into the correlation matrix so to know the level of association among the variables whiles changing all variables into the same unit. From the output,there are quite a good number of not so high correlations among the variables. Nevertheless, Outflow and Real Exchange Rate showed a considerably degree of positive correlation. Inflow and Outflow as they seem to rhyme showed quite an appreciable level of positive correlation also.With these and others, it is important to understand how these could adequately impact on a particular incidence without necessarily compromising estimation precision. To sidestep any issues of obtaining imprecise estimates in the near future, the need for factor analysis as a pilot study is therefore obvious.

The correlation matrix of the nine indicators is shown below.

	inf	ge	rer	ir	inflow	out.flow	export	import	gdp
inf	1.00000000	0.04107961	-0.1850347	0.1213513	-0.06556681	-0.1317890	-0.09639676	-0.07128051	0.16025417
ge	0.04107961	1.00000000	0.4084715	-0.4731129	0.44708003	0.4081610	0.16315973	0.40110873	-0.29846775
rer	-0.18503468	0.40847148	1.0000000	-0.7932240	0.77260556	0.8431808	0.78018634	0.79894362	-0.40268302
ir	0.12135128	-0.47311287	-0.7932240	1.0000000	-0.64750746	-0.6968585	-0.65862361	-0.77094538	0.12173194
inflow	-0.06556681	0.44708003	0.7726056	-0.6475075	1.00000000	0.8241653	0.62527326	0.68344220	-0.32721453
out.flow	-0.13178898	0.40816098	0.8431808	-0.6968585	0.82416528	1.0000000	0.73008270	0.75805287	-0.26746298
export	-0.09639676	0.16315973	0.7801863	-0.6586236	0.62527326	0.7300827	1.00000000	0.66967538	-0.08138682
import	-0.07128051	0.40110873	0.7989436	-0.7709454	0.68344220	0.7580529	0.66967538	1.00000000	-0.11717267
gdp	0.16025417	-0.29846775	-0.4026830	0.1217319	-0.32721453	-0.2674630	-0.08138682	-0.11717267	1.00000000

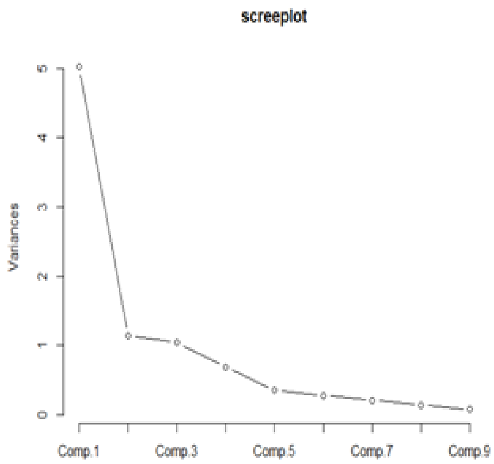
B. Principal component and factor analysis

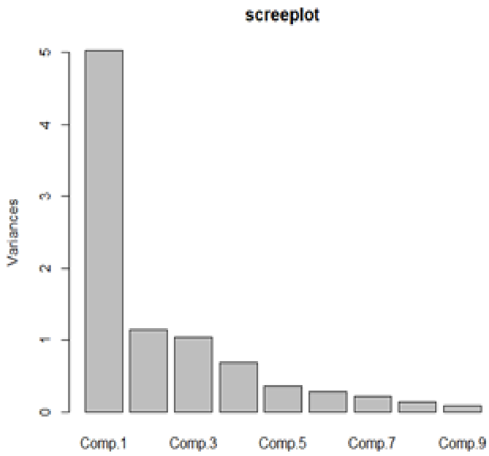
1. Principal component analysis

Prior to running the factor analysis on these indicators, the principal component of the indicators is ran to have a general idea of the number of factors we will be considering; decision is based on the number of components with standard deviation greater than or equal to one. The principal components from the sample correlation matrix of these macroeconomic indicators are shown below:

	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5	Comp.6	Comp.7	Comp.8	Comp.9
Standard deviation	2.2426255	1.0698871	1.0235262	0.8317394	0.60116102	0.53121937	0.46262422	0.38309884	0.28670851
Proportion of Variance	0.5588188	0.1271843	0.1164007	0.0768656	0.04015495	0.03135489	0.02378013	0.01630719	0.00913353
Cumulative Proportion	0.5588188	0.6860031	0.8024037	0.8792693	0.91942426	0.95077915	0.97455928	0.99086647	1.00000000

From the output, the cumulative proportion of variance explained by the first three principal components is roughly 80%. We generally cannot tell beforehand the number of factor loadings to consider by merely observing the cumulative proportion of variance explained by the components. For us to have a convincingand pictorial representation of the number of factors to consider from, we look at the scree plots of the principal components. The graphs below are the scree plots of the principal components of the nine macroeconomic indicators.





We can infer from the scree plots that, there are three factors in which these nine indicators can be classified given that three principal components have proportion of variance explained exceeding unity. More so, deducing from the line scree plot on the left, there is an L bow shape of line forthe first three principal componentsindicating that, these nine indicatorscan be grouped into three factors without losing much information.

2. Factor analysis

(i) The maximum likelihood method of estimation of factor loadings

Previously from the principal component analysis, it was ascertained that, we could group the nine indicators into three factors without losing a considerable amount of information. We can estimate the factor loadings using the maximum likelihood estimation method however scaling the data in order to achieve uniformity of units across the indicators. Output of the unrotated factor loadings, communalities and the specific variances from the maximum likelihood estimation method is given below:

The unrotated factor loadings (three factor model)

				communalities	specificvariance
inf	-0.09672078	-0.1660584	0.168525623	0.06533117	0.93466883
ge	0.36126342	0.3202336	0.679020513	0.69412969	0.30587031
rer	0.85734274	0.4504737	-0.067974235	0.94258364	0.05741636
ir	-0.83369725	-0.1676411	-0.157931514	0.74809702	0.25190298
inflow	0.73310241	0.3682344	0.095190642	0.68209696	0.31790304
out.flow	0.82919264	0.3135238	0.009307640	0.78594420	0.21405580
export	0.81982428	0.1257554	-0.258934618	0.75497341	0.24502659
import	0.84928510	0.1636814	0.086954887	0.75563793	0.24436207
gdp	0.05386347	-0.9960410	0.001516033	0.99500131	0.00499869

From the output, most of the variations in Real Exchange Rate, Interest Rate, Inflow, Outflow, Export and Import are explained by Factor 1. Factor 2 explains most of the variations in GDP while Factor 3 explains most of the variations in Inflation and Government Expenditure. Having a critical observation of the factor loadings, we realized the loadings for Inflation across the three factors did not differ considerably. This leaves us indecisive as to which factor among the three to classify inflation given the insignificant differences among the loadings.Given this, we resorted to the Varimax approach of rotation.

The rotated factor loadings (three factor model)

				communalities	specificvariance
inf	-0.13300438	-0.19005743	0.1073275	0.06533117	0.93466883
ge	0.29084577	0.09558367	0.7748562	0.69412969	0.30587031
rer	0.89433844	0.34772962	0.1477380	0.94258364	0.05741636
ir	-0.81542367	-0.02243696	-0.2875375	0.74809702	0.25190298
inflow	0.74218404	0.24284671	0.2688592	0.68209696	0.31790304
out.flow	0.84406326	0.20020748	0.1828069	0.78594420	0.21405580
export	0.85542840	0.08767802	-0.1246123	0.75497341	0.24502659
import	0.84022108	0.03445462	0.2201802	0.75563793	0.24436207
gdp	-0.03340886	-0.96410787	-0.2537344	0.99500131	0.00499869

After rotation, we observe a slight shift of extreme loading from factor 3 to factor 2 for Inflation (ignoring the sign).This implies a different factor classification for Inflation Rate after rotation. Following from this result, it implies Inflation and GDP can now be classified into factor 2. Factor 3 explains most of the variations in Government Expenditure as we observe a change of loading from 0.679 in the unrotated factor model to 0.775 in the rotated factor model. One advantage of the maximum likelihood estimation method in our case is the fairly low values of the specific variance. Aside inflation with a very large specific variance, all others have the values below 0.5.However, one lapse here is, in the maximum likelihood estimation method, the rotation did not very much present a clear picture of the extreme loadings on Inflation. So to classify Inflation into any of the three factors may not be obvious. With this, the principal component method could be useful.

(ii) The principal component method of estimation of factor loadings

Previously we estimated the factor loadings, communalities and the specific variances using the maximum likelihood estimation method, now we turn to the principal component method. The output for the rotated and the unrotated factor loadings are given below.

The unrotated factor loadings (three factor model)

				communalities	specificvariance
inf	-0.06950893	-0.41006365	0.7862388003	0.7911551	0.2088449
ge	0.23530846	0.29062365	0.5485188517	0.4407051	0.5592949
rer	0.42254018	0.05008110	-0.0819936592	0.1877713	0.8122287
ir	-0.38237459	0.14576862	-0.0023724694	0.1674644	0.8325356
inflow	0.38629289	0.04294301	0.0888955172	0.1589687	0.8410313
out.flow	0.40718647	-0.03864157	-0.0371666365	0.1686754	0.8313246
export	0.35665897	-0.28799589	-0.2207566287	0.2588807	0.7411193
import	0.38738386	-0.19895892	-0.0009225975	0.1896518	0.8103482
gdp	-0.15084102	-0.77315298	-0.1273142463	0.6367275	0.3632725

The rotated factor loadings (three factor model)

				communalities2	specificvariance2
inf	-0.09605654	-0.06602637	0.92238273	0.7911551	0.2088449
ge	0.34802061	0.67488385	0.33729289	0.4407051	0.5592949
rer	0.88972361	0.30902047	-0.14407216	0.1877713	0.8122287
ir	-0.85938546	-0.14201101	-0.03103773	0.1674644	0.8325356
inflow	0.79725037	0.35340724	0.01944575	0.1589687	0.8410313
out.flow	0.88132763	0.23834755	-0.05900750	0.1686754	0.8313246
export	0.87432271	-0.10430448	-0.10216515	0.2588807	0.7411193
import	0.88768659	0.09623305	0.05293466	0.1896518	0.8103482
gdp	-0.05539629	-0.86017958	0.26958029	0.6367275	0.3632725

Unlike the maximum likelihood estimation technique which is quite insensitive to the Varimax rotation in our case; we have realized from the principal component method in the unrotated factor model that extreme loadings were not observable as it becomes hard to classify the indicators into factors given the minor differences among the indicators across the three factors. However, after rotation as seen in the second output above, there is a significant shift of loadings. This enables us to easily tell which indicator belongs to which factor. Important point to note here is that communalities and the specific variances remain the same for the rotated and unrotated factor model. So rotation does not alter the factor loadings but convinces us beyond doubt that a particular indicator is classified into the right factor.

Comparison of the maximum likelihood and the principal component methods for the rotated factor loadings.

Variable	Maximum likelihood				Principal component			
	Estimated rotated factor loadings			Spec var.	Estimated rotated factor loadings			Spec var.
	F1	**F2**	**F3**		**F1**	**F2**	**F3**	
1. Inflation	-0.097	-0.166	**0.169**	0.935	-0.096	-0.066	**0.922**	0.208
2. Government Exp.	0.361	0.320	**0.679**	0.306	0.348	**0.675**	0.337	0.559
3. Real Exchange Rt.	**0.857**	0.450	-0.068	0.057	**0.890**	0.309	-0.144	0.812
4. Interest Rate	**-0.834**	-0.168	-0.158	0.252	**-0.859**	-0.142	-0.031	0.833
5. Inflow	**0.733**	0.368	0.095	0.318	**0.797**	0.353	0.019	0.841
6. Outflow	**0.829**	0.314	0.009	0.214	**0.881**	0.238	-0.059	0.831
7. Export	**0.820**	0.126	-0.258	0.245	**0.874**	-0.104	-0.102	0.741
8. Import	**0.849**	0.164	0.087	0.244	**0.887**	0.096	0.053	0.810
9. GDP	0.054	**-0.996**	0.002	0.005	-0.055	**-0.860**	0.270	0.363
Cumulative proportion of total (standardized) sample variance explained	0.466	0.647	0.714		0.613	0.694	0.802	

Comparison of the unrotated and rotated factor loadings of the principal component method

Variable	Principal component							
	Estimated unrotated factor loadings			Spec var.	Estimated rotated factor loadings			Spec var.
	F1	**F2**	**F3**		**F1**	**F2**	**F3**	
1. Inflation	-0.070	-0.410	**0.786**	0.208	-0.096	-0.066	**0.922**	0.208
2. Government Exp.	0.235	0.291	**0.549**	0.559	0.348	**0.675**	0.337	0.559
3. Real Exchange Rt.	**0.423**	0.050	-0.081	0.812	**0.890**	0.309	-0.144	0.812
4. Interest Rate	**-0.382**	0.146	-0.002	0.833	**-0.859**	-0.142	-0.031	0.833
5. Inflow	**0.386**	0.043	0.089	0.841	**0.797**	0.353	0.019	0.841
6. Outflow	**0.407**	-0.039	-0.037	0.831	**0.881**	0.238	-0.059	0.831
7. Export	**0.357**	-0.288	-0.221	0.741	**0.874**	-0.104	-0.102	0.741
8. Import	**0.387**	-0.199	-0.001	0.810	**0.887**	0.096	0.053	0.810
9. GDP	-0.151	**-0.773**	-0.127	0.363	-0.055	**-0.860**	0.270	0.363
Cumulative proportion of total (standardized) sample variance explained	0.559	0.686	0.802		0.559	0.686	0.802	

IV. DISCUSSION

The rotated (extreme) loadings of the principal component method of estimation are shown in red. Comparing these two estimation methods, it would be appropriate to use the principal component method given its relatively greater extreme loadings compared to the maximum likelihood estimation method. Using the principal component method it is explicit to say, Real Exchange Rate, Interest Rate, Inflow, Outflow, Export and Import can be classified into one group (Factor 1). Government Expenditure and GDP can also be classified into a different group (Factor 2) while the rate of Inflation in its own uniqueness constitutes a factor (Factor 3). Factor one tends to focus more on the Open Economy. This is because Real Exchange Rate, Interest Rate, Inflow, Outflow, Exports and Imports cannot be possible when the economy is a closed one except for the case of Inflation in some sense. For instance the indicators, Imports and Exports adequately define the openness of an economy. Assuming, Government Expenditure is solely domestic, then the second factor is more to the closed economy than to the open economy if net exports/imports are not factored into GDP.Inflation in its own uniqueness is a factor. Consequently, factor one can be interpreted as the *Open Economy Factor*, factor twoby contrast is the *Closed Economy Factor* and factor three in its uniqueness is seen as the *Inflation Factor*.

V. CONCLUSION

Based on the findings above, the nine macroeconomic indicators showed fairly a degree of unpardonable redundancy inthe correlation matrix. Preliminary to the factor analysis we found from the principal component that three factors will suffice. From this, the factor analysis was adopted to identify the variables that are likely to be classified together as one factor. Three factor model was used with evidences from the scree plots of principal components. It was seen that those three components roughly explains 80.2% of the total variation in the data. We found that six of these indicators includingReal Exchange Rate, Interest Rate, Inflow, Outflow, Exports and Imports showed a very high likeness and these were classified into one factorcalled the *Open Economy Factor*. Government Expenditure and GDP exhibited great similarities and were classified into another factorcalled the *Closed Economy Factor* but Inflation on its own uniqueness from the others was classified as *Inflation Factor*. In effect, these methods served as tools for effectively reducing and grouping variables into fewer factions with little loss of valuable information. Thisgives the essence of the principal component and the factor analysis.

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